# Master Thesis: Solving the Forward and Inverse Problem of the 2D-Helmholtz Equation using a Convolutional Neural Network

Context: This thesis is part of an interdisciplinary project called Minimally Invasive Robot-Assisted Computer-guided LaserosteotomE (MIRACLE) from the Department of Biomedical Engineering at the University of Basel. The goal of this project is to develop a minimally invasive robotic endoscope to cut bones with laser-light.

During tissue ablation with laser-light, an acoustic wave is emitted (Kenhagho et al., 2018). By measuring these emitted waves, the characteristic speed of sound in different tissue types allows for an inference of the surrounding tissue type. However, the classical numerical approaches to do so (Nahum et al., 2019) are time-consuming and not yet suitable for a real-time feedback system. Hence, we want to improve the computational time of our simulation of the *Helmholtz equation* using a *Convolutional Neural Network* (CNN).

Task description: The goal of this thesis is to simulate the behavior of sound waves for inference of the underlying tissue using a CNN. You will learn the basics of solving *partial differential equations* with *finite elements* to numerically compute a sound wave given by the *Helmholtz equation*, solving the so-called *forward problem.* You will use the resulting simulations as *ground truth data* to train a CNN, which may then be used to speed up the computations considerably. In a final step, you reconstruct the structure of a simulated 2D bone – also known as solving the *inverse problem* – in an increasingly realistic setup. Most of the task involves programming in one form or another.

| Setup | Medium and Source | Wave simulation |
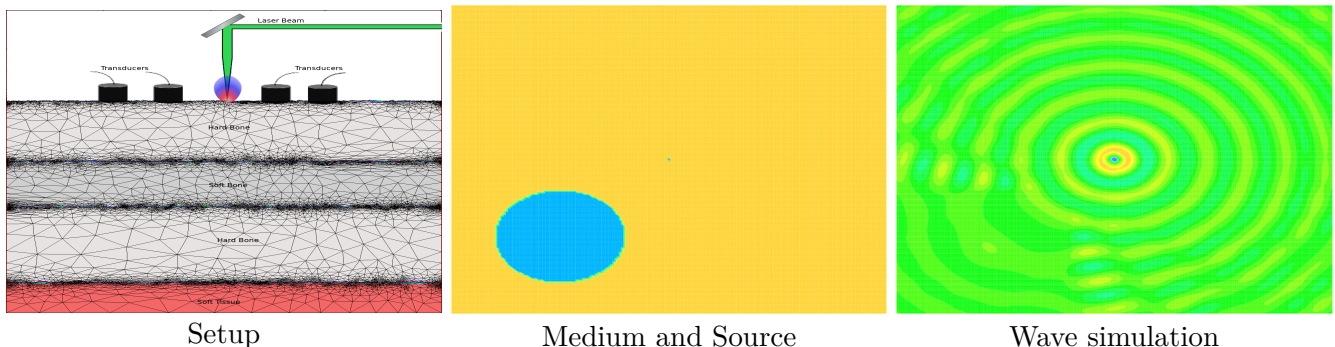
Workpackages:

- Preparation: Get a rough understanding on solving *partial differential equations* with the help of *finite elements* and the use of *convolutional neural networks* (CNN).
- Familiarisation of the setup: Get familiar with the existing code bases for the *finite element* approach and the CNN as well as with the programming languages they are written in (FreeFEM and Python).
- Data generation: Solve the *forward problem* of the *Helmholtz equation* with the approach of *finite elements*.
- Convolutional neural network: Use your generated data to solve the *forward* and *inverse problem* using a CNN.
- Evaluation: Analyse the limitations and constraints of the trained CNN.

Specific Requirements:

- Basic programming skills (any language)
- Good mathematical perception