A Principled Approach to Combining Inversion Recovery Images

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Introduction

Averaging images at different inversion times of an inversion recovery sequence can give a good contrast-to-noise ratio (CNR) between tissues. On the basis of the averaged magnetization inversion recovery acquisitions (AMIRA) [1] sequence we give a principled justification for such averaging. Using energy optimization, we show that uniform averaging, as is done in [1], is near-optimal and can only slightly be improved. As an example we optimize the CNR between different compartments in the spinal cord. Fig. 1 shows 10 fold upsampled exemplary inversion images of the AMIRA sequence of one axial cross-sectional tomogram on vertebra C4 level.

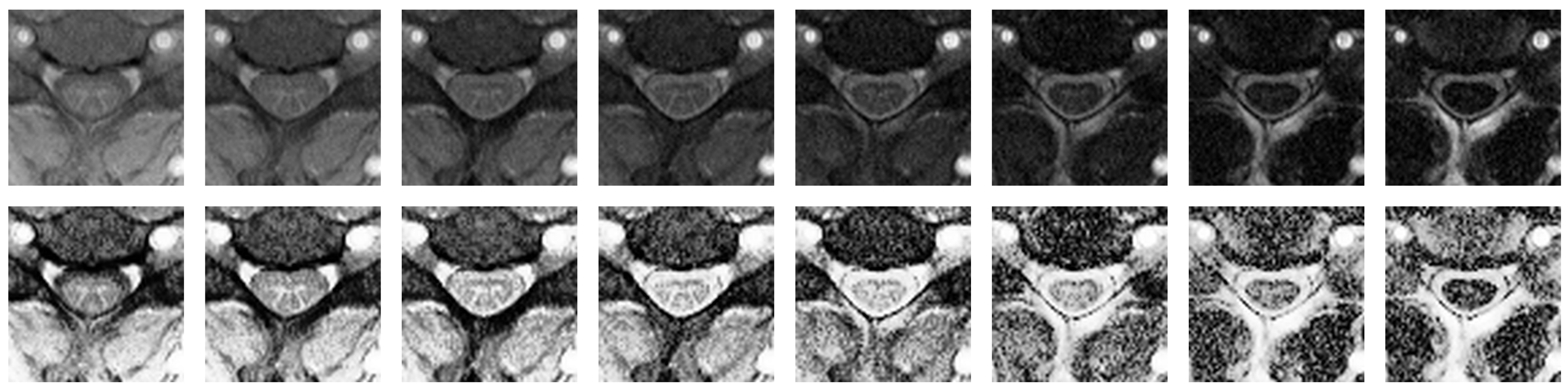

Fig. 1: Zoomed inversion images of the AMIRA sequence. *Top:* original, *bottom:* histogram equalized.

Fig. 2 shows optimized non-uniform averages next to the uniform averages.

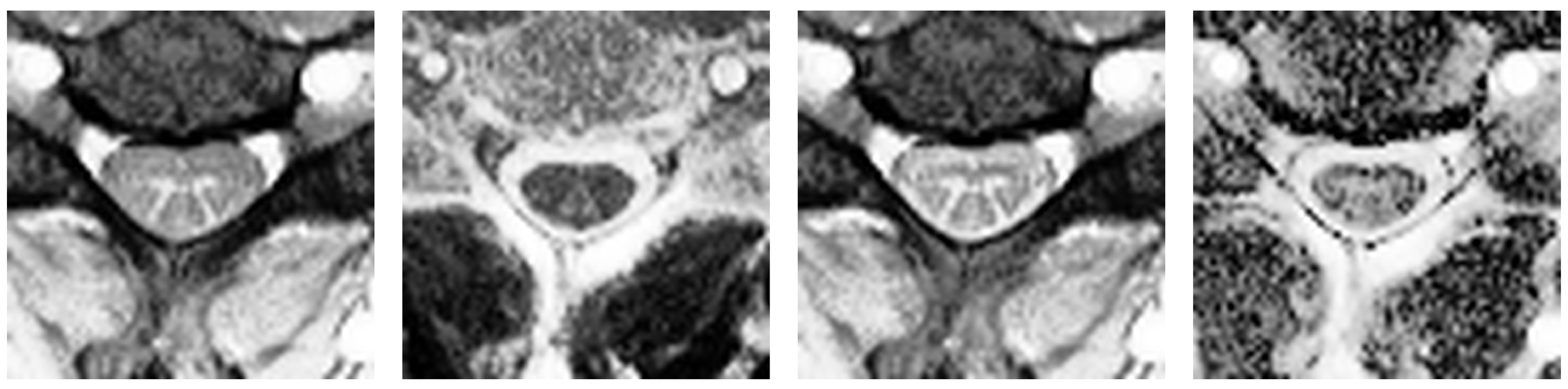

Fig. 2: *Left to right:* Optimized GM/WM and CSF/WM average, uniform averages of the first five and of the last three inversion images.

Method

Given images $I_1, \dots, I_n$ with values between 0 and 1, we want to find coefficients $c = (c_1, \dots, c_n)$ such that the linear combination $I_c := c_1 \cdot I_1 + \dots + c_n \cdot I_n$ has optimal contrast-to-noise ratio. Suppose we have a manual segmentation that differentiates between tissue A and B, and suppose tissue A has intensities close to 1 and tissue B intensities close to 0. To find the coefficients c, we optimize the energy

$$\begin{aligned} E(c) := {} & \lambda_1 \frac{\sum_{x \in A} (I_c(x) - 1)^2}{|A|} + \lambda_2 \frac{\sum_{x \in B} (I_c(x))^2}{|B|} + \\ & - \lambda_3 \big| E[I_c(A)] - E[I_c(B)] \big| + \lambda_4 V[I_c(A)] + \lambda_5 V[I_c(B)] + \\ & - \lambda_6 \sum_{k=1}^{n} c_k \big| E[I_k(A)] - E[I_k(B)] \big| + \lambda_7 \sum_{k=1}^{n} c_k V[I_k(A)] + \\ & + \lambda_8 \sum_{k=1}^{n} c_k V[I_k(B)] + \lambda_9 \left(1 - \sum_{k=1}^{n} |c_k| \right)^2, \end{aligned}$$

where $|A|$ is the area of A, and $E[I(A)]$ and $V[I(A)]$ are the mean and variance of the intensities of tissue A on image I, respectively. The 1st and 2nd summands in E force the linear combination I_c to be as close to the segmentation as possible, the 3rd negative term maximizes the contrast, and the 4th and 5th terms minimize noise. The terms 6, 7 and 8 also maximize contrast and minimize noise, but on level of the individual inversion images. The 9th term constrains the coefficients' absolute values to sum up to 1. λ_i are hyperparameters.

Experiments & Results

We optimized coefficients for a total of 68 slices on 4 different subjects at different vertebral heights (C2-C5) with empirically chosen λ_1 to λ_9 of 100, 100, 1, 1, 1, 10, 10, 10, 1000, respectively. For each slice we calculated coefficients $c_{\mathrm{GM/WM}}$ and $c_{\mathrm{CSF/WM}}$ for optimal contrast, shown in Fig. 3.

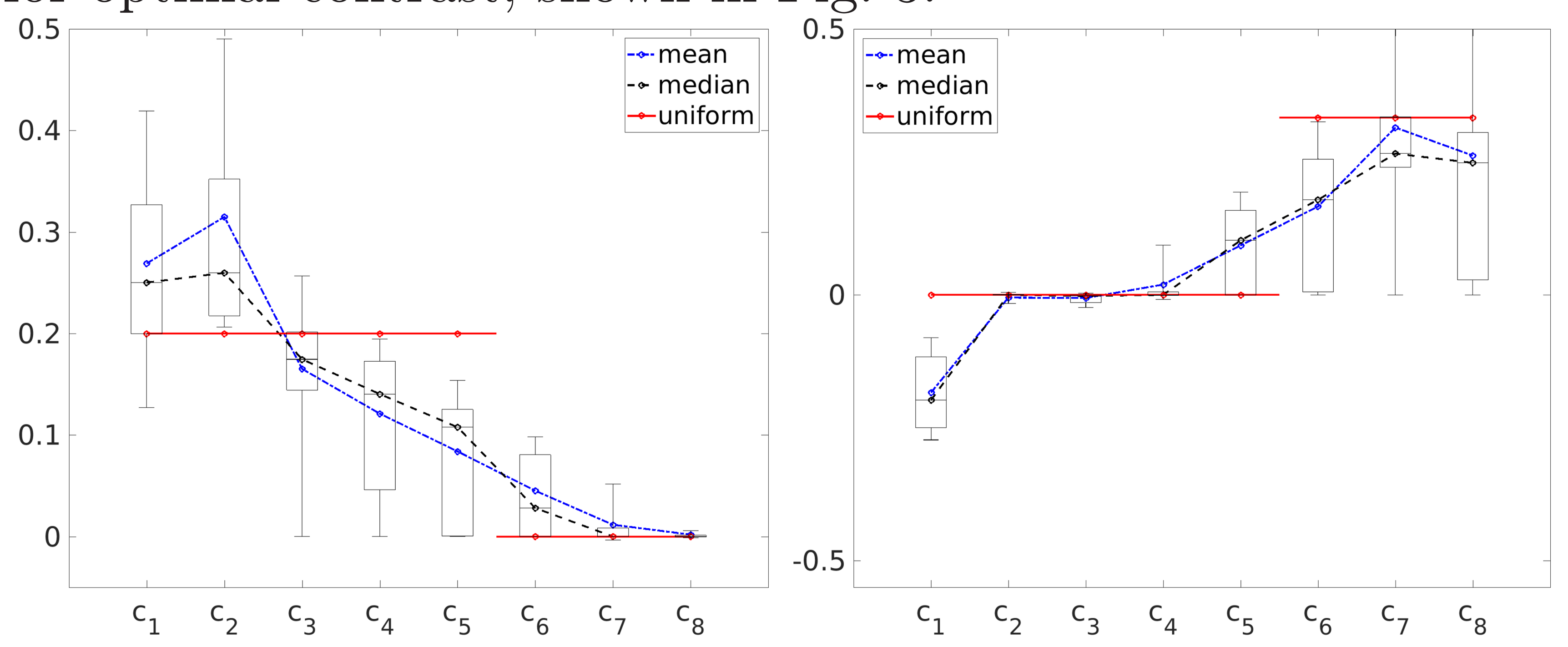

Fig. 3: Linear coefficients $c_{\mathrm{GM/WM}}$ (*left*) and $c_{\mathrm{CSF/WM}}$ (*right*). In red, the uniform averaging; in black, box plots of the optimized coefficients of all 68 slices with median, lower and upper quartile box, and 10th/90th percentile whiskers; and in blue, the mean values are shown.

Fig. 4 shows a quantitative comparison of CNR between uniform and proposed averaging. Optimized coefficients were calculated and evaluated for two cases: the optimal case (s), where the optimal coefficients c of each slice were used to calculate the combination I_c for each slice; and the global case (g), where the mean value coefficients of Fig. 3 were used for all slices.

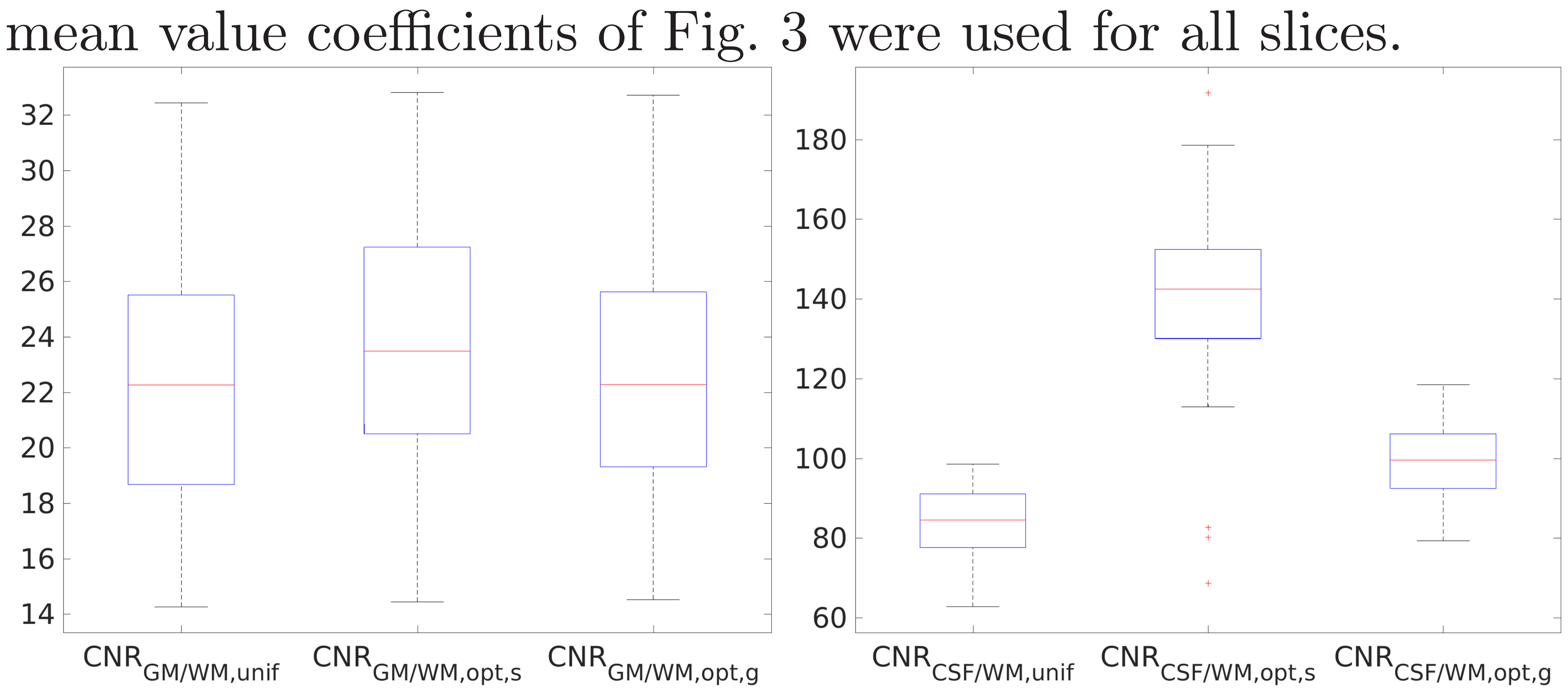

Fig. 4: CNR of the uniform averaging (unif) and the optimized (opt).

Tab. 1 shows a leave-one-subject-out cross validation. Each cross-validation uses the coefficients averaged over the optimal coefficients of the slices of the left out subject.

Left out subject	$\mathrm{CNR}_{\mathrm{CSF/WM,unif}}$	$\mathrm{CNR}_{\mathrm{CSF/WM,opt}}$	$\Delta\mathrm{CNR}_{\mathrm{CSF/WM,opt\text{-}unif}}$	$\mathrm{CNR}_{\mathrm{GMWM,unif}}$	$\mathrm{CNR}_{\mathrm{GMWM,opt}}$	$\Delta\mathrm{CNR}_{\mathrm{GM/WM,opt\text{-}unif}}$
1	81.98	91.87	9.88	24.99	25.36	0.37
2	90.26	101.57	11.31	24.66	25.64	0.98
3	87.37	102.44	15.07	22.71	23.65	0.94
4	72.64	88.15	15.51	17.37	18.07	0.70

Tab. 1: Leave-one-subject-out cross-validation. CNR mean values over all slices of the subjects that were not left out are shown.

Conclusion

With the proposed method we analyzed the uniform averaging technique of the inversion images of the AMIRA sequence. The found calculated coefficients are close to the uniform coefficients and the contrast-to-noise ratio can only slightly be improved. The hyperparameters λ_1 to λ_9 can be chosen for the needs, e.g. priorizing less noise or better contrast.

References

[1] M. Weigel and O. Bieri, "Spinal cord imaging using averaged magnetization inversion recovery acquisitions," Magn Reson Med, Jul. 2017.